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Instability Process in Low Reynolds Number Supersonic Jets

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Hot-wire measurements in low Reynolds number axisymmetric perfectly expanded cold jets of Mach number 1.4, 2.1, and 2.5 have been performed. There is a narrow band of unstable frequencies present in the instability of each jet which becomes broader, with more spectral peaks as the Mach number increases. The most dominant spectral component in each jet decreases in Strouhal number with increasing Mach number so that the Helmholtz number ($H = fd/a_j$) remains approximately constant at $H = 0.4$. The instability in each jet is initially in the flapping mode which is a superposition of right- and left-hand helices ($n = +1, -1$). Phase speeds of dominant spectral components calculated from frequency and wavelength measurements are between 58% and 70% of the jet exit velocity. No substantial alteration of the instability's structure appears associated with the transition from subsonic to supersonic instability waves (relative to the ambient acoustic velocity). The growth rates decrease strongly with increasing Mach number and are in reasonable agreement with the theories of Tam and Morris. The saturation and subsequent decay on the instability is coincident with a drastic decrease in the coherence of the instability. Associated research shows that the rapid growth and decay is of fundamental importance in the noise generation process.

Nomenclature

a_j	= local speed of sound
a_0	= speed of sound outside jet
c	= wavespeed in the downstream direction
D	= diameter of the jet
d	= effective diameter of the jet
f	= frequency
H	= modified Helmholtz number $= M \cdot St = fd/a_j$
k	= complex wavenumber
k_i	= imaginary portion of k
k_r	= real portion of k
M	= Mach number of the jet exit on the centerline
$\tilde{m}$	= normalized mass velocity fluctuations $= (\rho u)'_{\rm rms}/(\overline{\rho u})$
n	= azimuthal mode number
p_c	= test chamber pressure
r	= radial distance from jet centerline
$r(\alpha)$	= radial distance from jet centerline where $u = \alpha U_0$
Re	= Reynolds number $= \rho U d/\mu$, based on jet exit conditions
St	= Strouhal number $= fd/U$
t	= time
u	= local velocity
U	= mean centerline velocity of the jet at the nozzle exit
U_0	= mean centerline velocity of the jet at a given x location
x	= downstream distance from nozzle exit
y	= vertical distance from jet centerline
z	= horizontal distance from jet centerline
δ	= local shear layer thickness
τ	= period of the coherent structure
ω	= frequency
η	$= [r - r(0.5)]/\delta$
θ	= azimuthal angle
ϕ	= relative phase
ρ	= local density
μ	= viscosity
λ	= wavelength of fluctuation
$\overline{(\ )}$	= time averaged quantity
$(\ )'_{\rm rms}$	= room mean square of fluctuating quantity

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I. Introduction

A THOROUGH understanding of the character of the flow fluctuations in high-speed jets is essential to the development of a comprehensive theory of their noise generation. Motivated by the jet noise problem, the goal of our research is to gain an adequate understanding of the flow fluctuation processes in high-speed air jets. In particular, the present work reports on flowfield experiments with supersonic axisymmetric jets in the Mach number range from $M = 1.4$ to $M = 2.5$.

During the past several years a class of jet noise theories has undergone development by several authors including Tam,[1,2] Chan,[3,4] Liu,[5,6] and Morris.[7-9] These theories are built upon a substantial amount of experimental evidence which has demonstrated that there is an organized structure in turbulent free shear flows. It appears for supersonic jets (and to a lesser extent for subsonic jets) that the large-scale organized structure is of fundamental importance in the noise generation process.[10-12]

The most recent instability-type jet noise theory (Morris and Tam[9]) contends that the essential noise generating properties of a supersonic jet can be described by a quasilinear instability theory. In this theory, the growth and decay of individual spectral components are controlled by the development of the mean flow profiles.

For several years we have been making flow and acoustic measurements on supersonic jets operating at low Reynolds numbers.[12,13] The low Reynolds numbers are achieved by performing the experiments in an anechoic vacuum chamber where the ambient pressures are maintained at a controlled fraction of atmospheric pressure. Two major advantages of operating the jets at low Reynolds numbers are: 1) it enables hot-wire anemometry to be used; and 2) the small-scale turbulence which masks the coherent flow structure is decreased, making the structure's detection and characterization much easier.

Our previous work established that at Reynolds numbers near $Re=8000$, the flow fluctuations of an $M=2.2$ jet are conventional instability waves commonly found in flows undergoing transition from laminar to turbulent flow.[13] Acoustic measurements demonstrated that these instability waves radiate noise equivalent in strength to that produced by the noise generators of conventional, fully turbulent, high Reynolds number jets.[12] We also established that this remarkable fact is true in cases where the instability waves travel downstream subsonically with respect to the ambient air, thus precluding the possibility of eddy-Mach wave radiation in these cases.

The goal of the present research was to characterize the instability of supersonic jets of Mach numbers 1.4, 2.1, and 2.5 in the Reynolds number range around 8000. In this Reynolds number range the jet instability has its maximum coherence so that its properties, as well as the acoustic properties, can be most clearly identified. Growth rate, wavelength, and wave orientation of dominant spectral components of the instability for the three Mach number jets were measured in order to characterize the instability.

II. Experimental Apparatus

A. General Facility

The experiments were performed in our freejet test facility consisting of a low pressure anechoic chamber. Since the facility has been described in detail elsewhere,[12,14] only a brief description will be given here. The overall dimensions of the chamber are $0.7\times0.8\times1.2$ m and it is lined with 5-cm-thick acoustic absorption material to dampen the reverberant sound field. Upstream stilling chamber pressure and test chamber pressure are independently controlled to obtain perfectly expanded jets. In all experiments reported herein the nozzle exit pressure is maintained within 3% of the test chamber pressure.

Four different axisymmetric supersonic nozzles were used in these experiments; they were designed for Mach 2.5 with exit diameters of 6.9 mm and 9 mm, and for Mach 2.0 and 1.5 with exit diameters of 10 mm. The inviscid nozzle contours were calculated by a NASA computer program based upon the method of characteristics.[15] A boundary-layer correction computed using the method of Rott and Crabtree[16] was added to the inviscid contour. In this manner, each nozzle was designed for uniform parallel flow at the exit. The Reynolds number used in the design of each nozzle ($10,000\leq Re\leq 20,000$) was selected to be in the midrange of all experiments conducted at Oklahoma State University.

The stilling section at the inlet to the jet is 15 cm in diameter with a 5-cm section of foam, several perforated plates, a 7.6-cm-long honeycomb section, and six fine screens. A cubic contraction section is used to mate the stilling section to the nozzles. The stagnation temperature of the air jets was room temperature (approximately 294 K) and the air was dehumidified before being used in the facility.

For some of the measurements presented here the jet was artificially excited using a glow discharge excitation technique reported earlier.[13] As will be shown in the experimental results, the level of excitation was relatively low (particularly in comparison with the experiments of Chan[4] and Moore[17]) so that the behavior of the natural instability is not appreciably changed with its application.

B. Instrumentation

Mean flow measurements were performed in the jet using pitot pressure, static pressure, and hot-wire probes. A single probe was mounted on a three-direction traverse system with potentiometer readout to provide probe location. Details of the data reduction can be found in Morrison.[14]

Flow fluctuation measurements were made using a Disa 55 M constant temperature hot-wire anemometer system. The probes consist of Disa Model 55 A 53 subminiature probes epoxied to the upper edge of slender brass wedges. For all measurements reported herein the frequency response of the hot-wire system exceeded 40 kHz, which far exceeds the important frequency content of the jets. An active bandpass filter (4 pole Butterworth) with a lower cutoff frequency of 3 kHz was used in all hot-wire measurements to eliminate the signal caused by acoustic resonances of the test chamber.

Reduction of the hot-wire data has been accomplished using a technique similar to the one used by Johnson and Rose[18] and by Ko et al.[19] These techniques follow the original ideas of Kovasznay[20] and Morkovin.[21] The results of the hot-wire fluctuation measurements will be presented in terms of mass velocity fluctuations $(\rho u)'_{\rm rms}$ which are nondimensionalized with the local mean mass velocity ($\tilde{m}=(\rho u)'_{\rm rms}/\bar{\rho}\bar{u}$). Since $\tilde{m}$ contains both velocity and density fluctuations and their covariance, values of $\tilde{m}$ are substantially larger than $u'_{\rm rms}/\bar{u}$ estimates.[14]

A Tektronix Model 7L5 Spectrum Analyzer was used to measure spectra of individual signals. The spectrum analyzer used a constant bandwidth of 1 kHz (in most cases) and this bandwidth corresponds to $\Delta St=0.014$ for the spectra presented in this paper.

The relative phase between the exciter and the hot-wire was measured using a Saicor model SAI 43A correlation and probability analyzer by cross correlating the two signals. The Saicor analyzer was also operated in the enhance mode in order to phase average the hot-wire signal. The phase averaged signal is

$$\langle q(t)\rangle=\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N}q(t+n\tau)$$

where τ is the period of the coherent structure in the jet. In this manner the coherent structure can be extracted from the full signal and a measurement of the fraction of coherent structure contained in the full spectrum can be made.

III. Experimental Results

A. Jet Test Conditions

The majority of the measurements in the present study have been performed at one Reynolds number for each Mach number jet. This is in view of the fact that Behrens[22] showed, for the laminar far wake of a cylinder in supersonic flow, that the growth rates of the spectral components of the instability are only weakly dependent upon Reynolds number. The test Reynolds numbers are listed in Table 1 under the Basic measurements column. As shown in Ref. 14, the spectral content of the instability changes little with moderate changes in Reynolds number. The Reynolds numbers used here were chosen to maximize the identifiable flow instability with respect to the turbulence. At Reynolds numbers somewhat

Table 1 Jet test conditions

M	D, mm	Re	D/d	P_c, atm	U/d, s^{-1}
Basic measurements					
1.4	10	3700	1.07	0.0075	43,700
2.1	10	7900	1.06	0.0072	55,800
2.5	6.9	8700	1.12	0.0073	93,300
2.5	9	8700	1.12	0.0056	71,100
Additional measurements					
1.4	10	6400		0.013	43,700
Previous measurements[12,13a]					
1.3	8.03	5600			48,000
2.2[a]	6.35	14,400			105,000
2.3	6.35	10,000			102,000

[a] In these experiments the air was not dehumidified before being used in the jet facility. This resulted in a slightly lower Mach number than later measurements with the same nozzle using dried air.

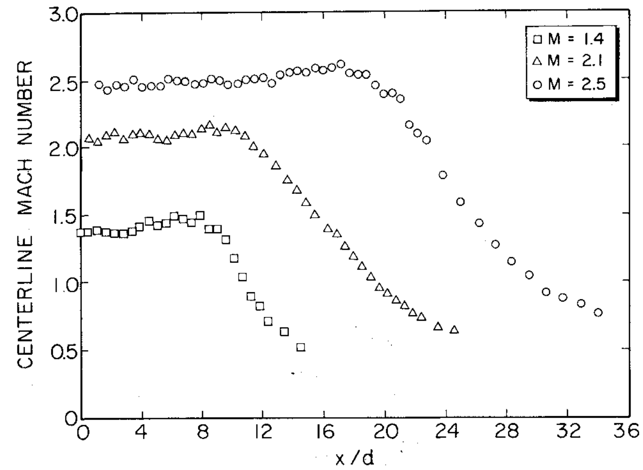

Fig. 1 Centerline Mach number distributions, $M = 1.4$, 2.1, and 2.5 jets.

lower than those used, the jets are completely stable. At Reynolds numbers an order-of-magnitude higher the shear layer close to the nozzle exit becomes unstable and the resulting turbulent masks the organized structure of the jet.

B. Mean Flow

Local Mach number is calculated from pitot pressure and static pressure measurements. Distributions of Mach number along the jet centerline were obtained for all three jets and are presented in Fig. 1. An obvious feature of these data is the dependence of the length of potential core on Mach number. As shown in Morrison,[14] this behavior is consistent with numerous other researchers' measurements in high Reynolds number supersonic jets (documented by Nagamatsu and Sheer[23]). However, the length of the potential core is also inversely dependent upon Reynolds number in the range of Reynolds numbers of the present experiments. The extended length of the potential core in the Mach 2.5 jet can be substantially attributed to the choice of a relatively low Reynolds number ($Re = 8700$) for this jet.

It is evident from the centerline Mach number distributions that there is a slight spatial oscillation of Mach number in the potential core region. No doubt this is a result of a slight deviation from parallel flow at the exit of the jet nozzles. (Recall that the boundary-layer corrections in the nozzle contour design were performed using a Reynolds number that is higher than that used in the present experiments.)

Earlier measurements reported in McLaughlin et al.[13] indicated that the weak wave cell structure has a controlling influence on the jet instability as was suggested by Tam.[1] Numerous additional experiments, including measurements performed in a Mach number 0.9 jet by Stromberg et al.,[24] have provided convincing evidence that the weak wave cell structure has little effect on the instability for nearly perfectly expanded jets.

Mach number profiles computed from pitot and static pressure probe measurements were also recorded for all jets. These profiles are documented in Morrison.[14] Hot-wire recovery temperature measurements also presented in Ref. 14 established that the local stagnation temperature in the jet is approximately uniform and equal to the upstream reservoir temperature. Consequently, local velocity can be calculated directly from the Mach number measurements. Mean velocity data for the $M = 2.5$ jet are presented in Fig. 2. (The $M = 1.4$ and 2.1 data are similar.)

Collapse of the mean flow data onto a common curve is accomplished by plotting local velocity, nondimensionalized with the local centerline velocity U_0, as a function of the nondimensional variable $\eta = [r - r(0.5)]/\delta$. Downstream of the end of the potential core $r(0.5) = \delta/2$ and hence $\eta \geq -0.5$.

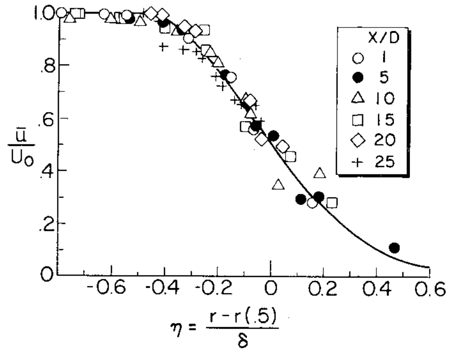

Fig. 2 Mean velocity data, $M = 2.5$ jet.

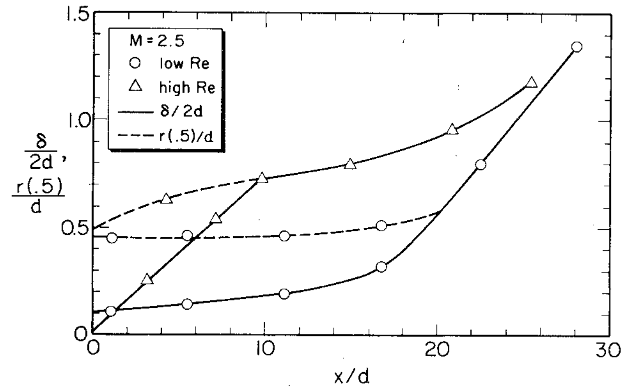

Fig. 3 Variation of mean velocity profile parameters.

As was the case in the subsonic jet data of Laufer et al.,[25] the mean velocity data for all three Mach numbers are reasonably well represented by a patched half-Gaussian profile given by:

$$\bar{u}/U_0 = \exp[-2.773(\eta + 0.5)^2] \quad \text{for } \eta > -0.5$$

$$= 1 \quad \text{for } \eta \leq -0.5$$

Such a representation has also been used by Morris and Tam[9] to fit a substantial amount of conventional high Reynolds number supersonic jet data.

Figure 3 shows how the shear layer half thickness parameter $\delta/2$ varies with downstream distance for the $M = 2.5$ jet in this study. This figure clearly indicates that the break point (the location where the jet begins to widen at a faster rate) is just upstream of the end of the potential core as determined by the data of Fig. 1. Behrens and Ko[26] and Sato and Okada[27] found that, in two-dimensional and axisymmetric wakes, the break point indicates the onset of nonlinear effects which cause the Reynolds stresses to diffuse the mean flow more rapidly and to generate random turbulence. Hot-wire measurements presented later in this paper conform to this observation.

Conventional high Reynolds number supersonic jets develop quite differently from the low Reynolds number jets of the present study in terms of the parameters describing the mean flow. Figure 3 also shows the two parameters $\delta/2$ and $r(0.5)$ for both low and high Reynolds number jets of Mach number 2.5. The low Reynolds number data came from the present study while the high Reynolds number data came from the empirical curve fits of Morris and Tam[9] using the centerline velocity distribution from Warren.[28]

In comparison with the high Reynolds number jet, the shear layer of the low Reynolds number jet grows much more slowly originally and leaves the jet exit with a thicker initial value. In the upstream region the low Reynolds number jet is laminar, which accounts for the low shear layer growth rate. Near the

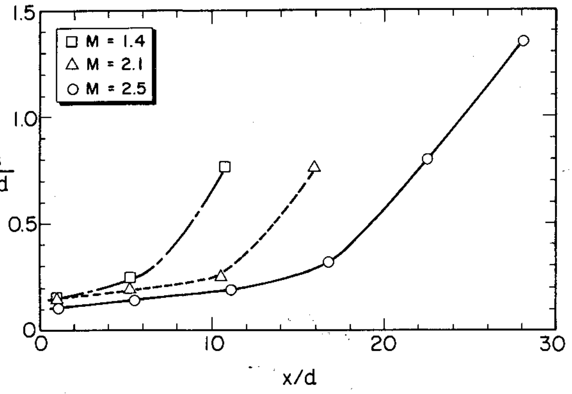

Fig. 4 Variation of shear layer half-thickness parameter with downstream distance.

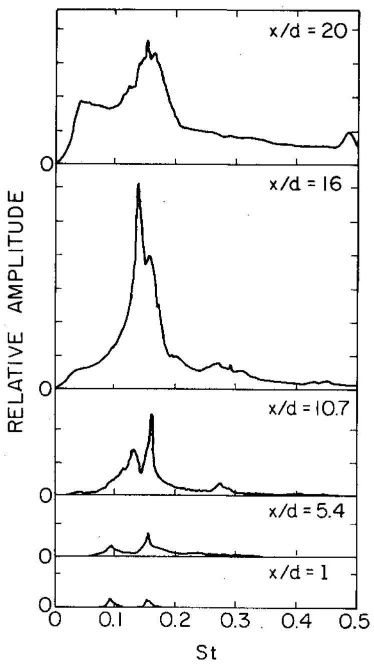

Fig. 5 Natural flow fluctuation spectra, $M=2.5$ jet.

end of the potential core, the instability waves are of sufficient magnitude to influence the mean flow and the shear layer and jet width grow very rapidly. This is the break point referred to above. Hot-wire fluctuation measurements, presented in the next section, present a clearer, more quantitative description of this phenomenon. The velocity profile parameters for the low Reynolds number $M=1.4$ and 2.1 jets have similar shapes as the $M=2.5$ jet. Figure 4 shows that the break point discussed above occurs just downstream of the end of the potential core as determined by the data of Fig. 1.

C. Spectral Content of Hot-Wire Fluctuations

Figure 5 shows hot-wire fluctuating voltage spectra for the Mach number 2.5 jet at several axial locations at the radial location of maximum fluctuations. As has been previously observed,[12,13] there are several dominant frequencies naturally present in a relatively narrow band from $St=0.1$ to 0.2. As the flow progresses downstream from the exit of the jet, different frequencies are found to dominate the spectra. Downstream of $x/d=17$ the spectra become much broader. However, the $St=0.16$ spectral component is the only one maintaining a discrete peak from $x/d=1$ to 20. We therefore consider it to be the dominant spectral component of the Mach number 2.5 jet.

At $x/d=20$ the flow fluctuation spectrum is relatively broad yet still contains a peak at $St=0.16$ and a substantial harmonic at $St=0.48$. The broadening of the spectrum and the production of the harmonic are indicative of nonlinear effects. Since this location is downstream of the break point, both mean and fluctuating measurements indicate an onset of nonlinear effects occurring near the end of the potential core.

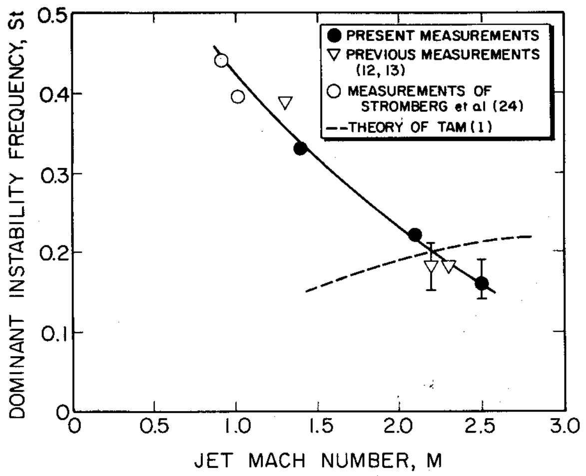

Fig. 6 Variation of dominant instability frequency with jet Mach number.

The lower Mach number jet exhibit the same spectral behavior as described above. The only substantial difference is a decrease in the number of large amplitude spectral components as the Mach number decreases.

As mentioned earlier, $St=0.16$ is considered the dominant spectral component of the $M=2.5$ jet, and, in a similar manner, the $St=0.22$ and $St=0.33$ components are considered the dominant ones in the $M=2.1$ and $M=1.4$ jets, respectively. (For more details on this see Morrison.[14]) The Strouhal number of the spectral component most preferred by each jet decreases with increasing Mach number, as shown in Fig. 6. It is noteworthy that Chan[3] found the same behavior for helium jets, but that the theory of Tam,[1] which includes a weak wave cell frequency selection mechanism, predicts contradictory behavior. Included on Fig. 6 are the dominant frequencies of low Reynolds number $M=0.9$ and $M=1.0$ jets measured in the same facility as the present study.[24] The frequency of the dominant instability component exhibits continuous behavior from the supersonic to the subsonic flow regime. This indicates that the weak wave cell structure of the nearly perfectly expanded supersonic jet, which is not present in the subsonic jet, has little bearing on the frequency selection process of the instability.

The Strouhal number of the dominant instability has a strong Mach number dependence. However, the modified Helmholtz number $H=St\cdot M=fd/a_j$ is a constant ($H=0.43\pm7\%$) for the dominant instability in all of the low Reynolds number jets from $M=0.9$ to 2.5. Powell[29] has discussed a similar trend in the Strouhal number for the noise radiated by subsonic jets. At this time, no satisfactory physical description has been offered to explain the observed result.

D. Growth of Flow Fluctuations

The growth of the instability is determined by measuring the fluctuation level (at the radial location of maximum fluctuations) as a function of downstream distance. Figure 7 shows the results of such measurements with the data reduced in terms of mass velocity fluctuations ($\tilde{m}$). Included in this figure are data from a naturally excited jet as well as from a jet artificially excited at three frequencies previously shown to be unstable ($St=0.14$, 0.16, and 0.18).

For the natural jet the level of fluctuations is initially less than 0.5% and grows approximately exponentially for the first 14 diameters of the jet (as shown by the solid straight line

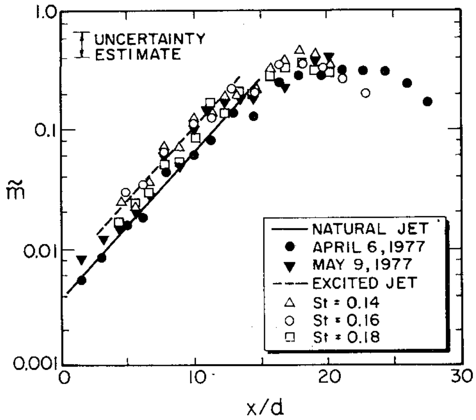

Fig. 7 Axial distribution of peak mass velocity fluctuation, $M=2.5$ jet.

on the semi-log plot). The data from the jets excited at three frequencies show a slight increase in the initial level of fluctuations, but the growth rate of the excited jet is the same as the natural jet case (as can be seen from the slope of the dotted line through the data). Using a linear stability type representation,‡ the growth rate is $-k_i d = 0.29 \pm 0.04$, where the latter number is the 95% confidence interval of the determined growth rate.

In both natural and excited jets the level of fluctuations saturates at about 30% of the local mean and then begins to decay. The fluctuation amplitude saturates near the axial location of the end of the potential core. The saturation and decay of the instability are consistent with the effect of the changing mean flow profiles (see Morris[8] and Morris and Tam[9]) predicted by quasilinear analyses. It is also interesting to note that the wave evolution data of Fig. 7 are quite similar to the acoustic source distributions measured by Laufer et al.[30] in supersonic jets.

Extensive growth measurements of numerous spectral components are documented in Morrison.[14] These results show that the growth rates of the dominant spectral components of the instability are almost the same as the growth rate of the full spectrum of flow fluctuations. Figure 8 depicts the variation of initial instability growth rate with Mach number (for the dominant spectral components in each jet). Included on the figure are growth rate predictions of Tam[1] and Morris[7] as well as data obtained earlier using jets with simple conical nozzles.[12,13] As shown by Morris[7] and Morris and Tam,[9] the growth rates are a strong function of mean flow profiles. As their analyses would lead us to expect, the growth rates of Tam's theory should be on the high side since that analysis uses top hat mean velocity profiles. Also, the data taken from Morris[7] are for the initial velocity profile of an isothermal jet (where the stilling chamber temperature is 1.4 times the ambient temperature). In view of these differences in the mean flow conditions, there is reasonably good agreement between the experiments and the theories; this is encouraging evidence of a basic capability of the stability analyses.

The growth rates depicted in Fig. 8 are shown to have the strong Mach number dependence (increasing stability with increasing Mach number) which has been established for many years.[31] As shown by Morrison,[14] there is also a moderate Reynolds number dependence on the instability growth rates, perhaps caused by changes in the mean flow

‡A linear stability analysis assumes a representation of the flow disturbances $Q(x,r,\theta,t)$ in the form: $Q(x,r,\theta,t) = \hat{Q}(r)$ $Re\{\exp[i(k_r x - \omega t + n\theta) - k_i x]\}$, where Re stands for the real part of { }. Since in the potential core region the mean mass velocity difference across the shear layer $\Delta\bar{\rho u}$ does not change with downstream distance, the initial growth rate of the instability $-k_i d$ is the initial slope of a $\log_e \tilde{m}$ vs x/d plot (see Ref. 22).

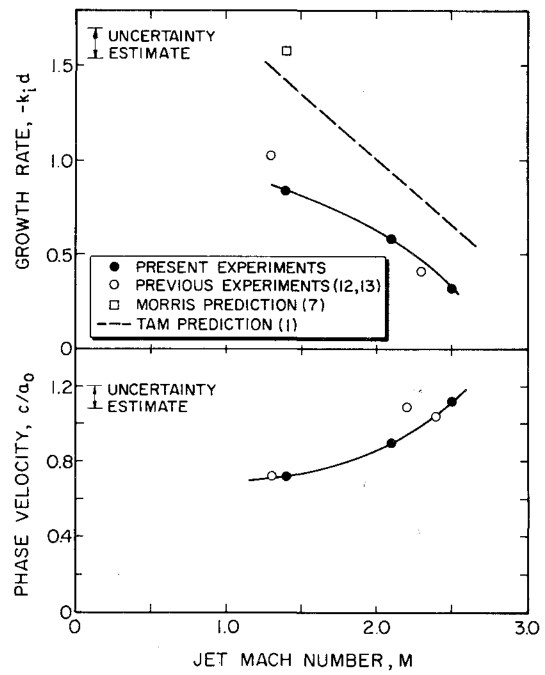

Fig. 8 Growth rate and phase velocity of the dominant instability component as a function of jet Mach number.

profiles. For this reason, all of the experimental data presented in Fig. 8 are in the Reynolds number range $Re = 6400\text{-}8700$. These are the only data in this paper which use the $Re = 6400$, $M = 1.4$ jet condition noted in Table 1.

As mentioned earlier, the saturation and decay of the instability can be attributed to changes in the mean flow profiles. On the other hand, such saturation is typical of nonlinear spectral interaction phenomena, as demonstrated by Miksad.[32] In the present experiments, the level of coherent structure characterizing the instability decreases rapidly, beginning at the axial location of maximum fluctuations (the saturation point). As shown in Morrison,[14] the fraction of organized fluctuations experiences a dramatic drop near the end of the potential core. Accompanying the drop in coherent structure is a very rapid increase in the broad spectrum fluctuations. No doubt a substantial fraction of the energy in the large-scale coherent instability is being transferred into a broad range of fluctuation scales and frequencies. Such interaction between spectral components is typical of nonlinear phenomena. Our earlier discussion pointed out that the spectral broadening occurs at the same locations in the jet as does the rapid increase in the shear layer growth. Since both phenomena are symptomatic of nonlinear interactions, it is unlikely that a pure linear theory will predict accurately the complete instability behavior.

The amplitude of the mass velocity fluctuations in the region of maximum fluctuations is approximately $\tilde{m} = 0.3$ for the $M = 2.5$ jet. Morrison[14] has performed a more extensive modal decomposition of the hot-wire fluctuations assuming that the pressure fluctuations have negligibly small influence on the response of the hot wire. With this assumption Morrison estimated that the peak velocity fluctuations u'_{rms} normalized with the jet exit velocity U are approximately $u'_{rms}/U = 0.10$ to 0.13 for the $M = 1.4\text{-}2.5$ low Reynolds number jets. The data of Fig. 7 together with similar data from other supersonic jets[14] indicate that the onset of nonlinear effects occurs at approximately 70% of the peak fluctuation level. This suggests that the onset of nonlinear effects occurs where $u'_{rms}/U = 0.08$, which appears to exceed by about a factor of two or three the linearity limit measured in unbounded subsonic free shear flows.[32,33] The difference in the flowfield and Mach number are offered as possible explanations for the apparent difference in linearity limit.

E. Relative Phase Measurements of the Dominant Spectral Components of the Instability

The wavelength and orientation of most of the dominant spectral components of the instability were determined by measuring the relative phase ϕ between the hot-wire and the excitation input signal. In this manner the axial wavenumber k_r and the azimuthal mode number n were determined for specific components in each jet.

The axial distribution of the relative phase for the three spectral components of the $M=2.5$ jet are recorded in Morrison.[14] Since these data are similar to axial phase distributions presented in Refs. 12 and 13 they are not repeated here. The wavelengths extracted from these phase distributions are presented in Table 2.

The wavelength of each spectral component remains approximately constant for at least 13 diameters of the jet ($x/d=7$-20). Since the wavelength and frequency of the disturbances are known, then the speed at which they travel in the downstream direction, c, can be calculated ($c=\lambda f$). The average value of c is also presented in Table 2. In the $M=2.5$ jet all three spectral components are traveling downstream supersonically at about 0.68 times the exit velocity U, and 1.2 times the ambient acoustic velocity a_0.

Similar relative phase measurements were made with the $St=0.33$ and 0.22 components of the $M=1.4$ and $M=2.1$ jet, respectively. The results of the wavelength and phase velocity measurements for all three jets are shown in Table 2. In addition, the phase velocities determined from these measurements are plotted in Fig. 8. Included as before are the data from previous experiments with more simply designed nozzle contours.[12,13] Note that for a jet Mach number of approximately 2.2 the wave speed of the instability becomes supersonic relative to the ambient acoustic velocity. There appears to be no substantial alteration of the instability phenomenon associated with the transition from subsonic to supersonic instability waves. Also, the major instability properties of the jets are not affected appreciably by the details of the nozzle or by moderate changes in the jet Reynolds number.

Attention is drawn to the instability phase speeds c, normalized with the jet exit velocities U, in Table 2. These phase velocities appear low, in comparison with the convection velocity measured by Dutt[34] in a Mach number 2.0 high Reynolds number jet. However, his technique is probably measuring a velocity close to the group velocity of the "wave system" which includes higher spectral components than experienced in our low Reynolds number jets. Recent measurements of Troutt[35] in a moderate Reynolds number supersonic jet demonstrate that the higher frequency components have phase velocities close to the convection velocity measured by Dutt, while the lower frequency components have phase velocities similar to those measured in the present investigation. This appears to explain the differences between the present measurements and those of Dutt. In addition, the measurements of Moore[17] in a subsonic jet, and the theory of Morris[8] for a supersonic jet show that the phase speed depends on frequency in a manner consistent with the present measurements.

Table 2 Axial wavelengths and phase velocities for $M=1.4$, 2.1, and 2.5 jets

M	St	λ/d	k_rd	c/a_0	c/U
1.4	0.33	1.83 ± 0.16	3.43	0.72	0.61
2.1	0.22	2.65 ± 0.11	2.37	0.89	0.58
2.5	0.14	4.69 ± 0.07	1.34	1.15	0.69
2.5	0.16	4.19 ± 0.06	1.50	1.11	0.67
2.5	0.18	3.81 ± 0.18	1.65	1.12	0.68

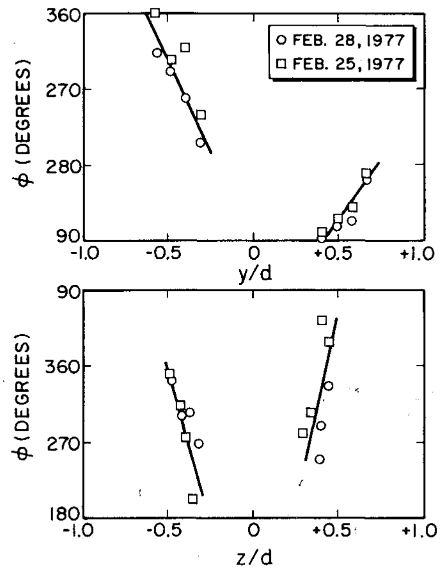

Fig. 9 Radial phase distribution, $St=0.16$, $x/d=9$, $M=2.5$ jet.

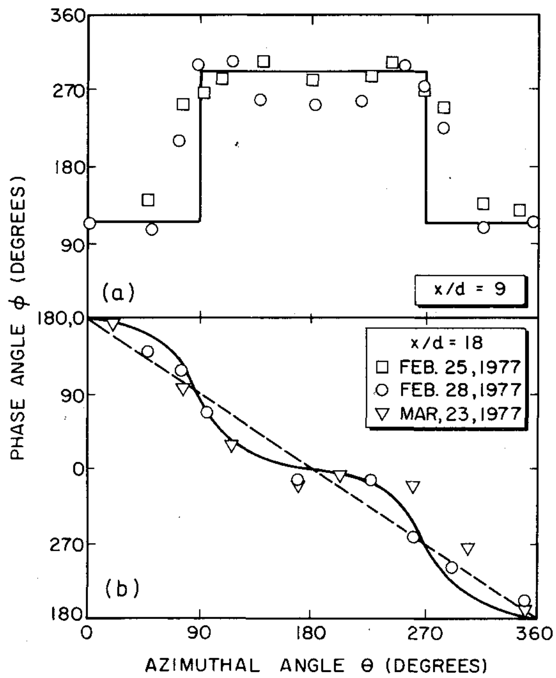

Fig. 10 Azimuthal phase distributions, $St=0.16$, $M=2.5$ jet.

Relative phase measurements were also made with the hot-wire probe referenced to the excitation signal in the radial and azimuthal directions in the jets. Figure 9 presents radial phase distributions for the $St=0.16$ component in the $M=2.5$ jet at an axial location of 9 jet diameters. These measurements show that there is a rapid change in phase across each shear layer and that there is symmetry about the vertical (y) axis and asymmetry about the horizontal (z) axis. Azimuthal phase measurements of the same spectral component were made at the radial location of maximum fluctuations in the shear layer and are plotted in Fig. 10a. The symmetry and asymmetry of the radial phase measurements, together with the azimuthal phase distributions, are consistent with a sinusoidal instability mode where the jet is flapping up and down. The flapping mode results from the superposition of two equal amplitude, phase locked helical waves $e^{i(k_rx+\theta)}$ and $e^{i(k_rx-\theta)}$ which are often referred to as the $n=+1$ and $n=-1$ modes.

The theoretical azimuthal phase distribution for the flapping mode is shown by the solid line in Fig. 10a. The data show a smoother 180 deg phase shift at the $\theta = 90$- and 270-deg points than the theoretical distribution. As dicussed by Morrison,[14] this is possibly due to the lack of precise probe resolution with the rapidly varying radial phase variation, although some higher azimuthal modes are no doubt present as well. It also should be pointed out that the flapping orientation is consistent with the single point exciter which is located on the top of the nozzle (on the y axis).

Returning our attention to the rapid phase changes across the shear layers shown in Fig. 9, it is noteworthy that Browand[33] and several other researchers have also observed a 180 deg shift across a two-dimensional, incompressible free shear layer. This type of phase shift in the disturbance yields the familiar cat's eye streamline pattern in a coordinate frame traveling at the phase velocity (see Betchov and Criminale[36]).

Radial and azimuthal phase distributions were also measured at $x/d = 13$ and $x/d = 18$ in the $M = 2.5$ jet. At $x/d = 13$ the same results were obtained as at $x/d = 9$. However, at $x/d = 18$ the azimuthal distribution changed to that shown in Fig. 10b. A cursory examination of the phase data suggests that the instability has transformed into a simple helix ($n = -1$). An $n = -1$ mode phase distribution is shown by the dotted line in Fig. 10b. However, more careful analysis suggests that the phase data result from an instability composed of unequal-amplitude right- and left-hand helices. Following the technique developed by Troutt,[35] a phase distribution can be obtained for a modal decomposition of the form:

$$Q(\theta) = A_l e^{i\theta} + A_{-l} e^{-i\theta}$$

The solid curve in Fig. 10b is the phase distribution for $A_{-l}/A_l = 1.5$. The reasonable agreement between this phase distribution and the one measured in the jet suggest that, at 18 jet diameters downstream of the exit, the instability has transformed from two equal-amplitude right- and left-hand helices (the flapping mode) to a superposition of unequal-amplitude right- and left-hand helices. It should be recalled that the level of coherent structure decreases rapidly downstream of the potential core where the transition from the flapping mode to the unequal-amplitude helices occurs. Both the left- and right-hand helices which make up the flapping mode are losing coherence. We suspect that a point is reached where the right-hand helix has lost more coherence in comparison with the left-hand one, perhaps because it was slightly weaker at the nozzle exit and throughout the jet. This would explain the phase distribution found at $x/d = 18$ shown in Fig. 10b. It is interesting to note that Hama et al.[37] obtained very similar phase distributions for the instability of the wake of a slender body at low speed. In fact, they also observed the same transformation from a sinusoidal to mixed helical modes as was found in the present study.

Measurements of the azimuthal phase distribution for the $St = 0.14$ and 0.18 components were also made. Both of these components possess a flapping mode at $x/d = 13$ which transforms into unequal amplitude left- and right-hand helices where the level of coherent structure decreases. These measurements, along with the axial phase measurements, indicate that all of the spectral components are initially a sinusoidal (flapping) mode that disintegrates at a location near the end of the potential core. The waves all travel downstream at approximately 68% of the mean exit velocity of the jet.

Since the $St = 0.14$, 0.16, and 0.18 spectral components have the same wavefront shape (helical) and similar phase velocity and growth rate (Morrison[14]), a proper description of the phenomenon is that the spectral components are part of the same instability mode. Some weakly nonlinear mechanism, possibly involving an acoustic feedback, is responsible for the multi-peaked behavior shown in the spectra of Fig. 5.

Similar radial and azimuthal phase measurements were performed with the $St = 0.33$ and 0.22 components of the $M = 1.4$ and $M = 2.1$ jets, respectively. Both jets possess a flapping instability at $x/d = 7.5$ with similar phase distributions as found in the $M = 2.5$ jet. The Mach 2.1 jet's instability transforms into unequal amplitude helices by $x/d = 14$ (this is just downstream of the end of the potential core). However, the instability in the Mach 1.4 jet lost its coherence before a transformation was measurable. Details of these experimental results can be found in Morrison.[14]

The most recent instability analysis of Morris and Tam[9] suggests that both the helical ($n = \pm 1$) and the axisymmetrical ($n = 0$) instability modes are present in the $M = 1.5$ jet, although they predict the $n = \pm 1$ mode to be more unstable. Using the point excitation technique we have been able to detect only the $n = \pm 1$ mode. This technique may tend to excite the $n = \pm 1$ mode preferentially. However, the instability is naturally present in the jet, and the exciter serves primarily to eliminate its natural phase randomness. Consequently, we believe that at the Reynolds number and mean flow conditions of the present experiments, the $n = \pm 1$ is the most unstable mode. This belief is reinforced by the ability of the same point exciter to excite both $n = \pm 1$ and $n = 0$ modes in a low Reynolds number, Mach 0.9 jet[24] and in a moderate Reynolds number, $M = 2.1$ jet.[35] In the supersonic jet at higher Reynolds numbers the annular shear layer near the exit becomes unstable. It is reasonable to expect the $n = 0$ instability to be unstable in this region and to prevail in the developing region of the jet. Such conjecture appears to be borne out by measurements of Dutt[34] and Troutt[35] in higher Reynolds number Mach 2 jets which indicate the presence of $n = 0$, $n = \pm 1$ and higher modes.

IV. Conclusions

Mean flow and mass velocity fluctuation measurements have been performed in low Reynolds number Mach 1.4, 2.1, and 2.5 jets. There is a narrow band of unstable frequencies present in the instability of each jet. This band is very narrow for the Mach number 1.4 jet and becomes wider (with several discrete peaks) as the Mach number increases. In terms of nondimensional frequency (Strouhal number) the frequency of the dominant mode decreases with increasing Mach number so that its modified Helmholtz number is approximately constant at $H = 0.4$.

The mode of the instability in all three jets is initially the $n = +1$ and -1 waves occurring simultaneously (resulting in the jet flapping up and down). In the Mach number 2.1 and 2.5 jets the instability was observed to transform into unequal amplitude right- and left-hand helices just downstream of the end of the potential core. This transformation which occurs where the flow fluctuations maximize, is a consequence of the substantial decrease in the coherence of the instability. The instability in the Mach number 1.4 jet lost its coherence before such a transformation could be observed.

The wavelength of the dominant instability was measured in each jet and the resulting phase velocity calculated. The wave velocity in the axial direction is between 58% and 70% of the jet exit velocity which is in substantial agreement with the theory of Morris[8] and experiments of Chan.[4] In terms of the ambient acoustic velocity the waves become supersonic at a jet Mach number of approximately 2.2. There appears to be no substantial alteration of the instability phenomenon associated with the transition from subsonic to supersonic instability waves.

The instability grows approximately exponentially from the jet exit to the end of the potential core. The growth rates decrease substantially with increasing Mach number, an instability property which has long been established.[31] Comparison of the growth rates with the theories of Tam[1] and Morris[7] yields reasonable agreement in view of established effects of the mean flow profiles.

Finally, the end of the potential core is identified as the region of maximum fluctuations and of drastic changes in the nature of the instability. Based upon the results of previous flow fluctuation and acoustic measurements we proposed that the strong growth, saturation and decay of the dominant instability is the major noise generation mechanism in the low Reynolds number supersonic jets.[12] (The first suggestion that such a process might be a substantial noise generation mechanism appeared in Mollo-Christensen.[38]) The noise generation properties of the three jets studied in detail here are documented in Morrison[14] and Morrison and McLaughlin.[39] It is our contention that increased understanding of the low Reynolds number instability process and its associated radiated noise will help in the analysis of conventional high Reynolds number jet noise production where the processses are decidedly more complicated. Subsequent work in our laboratory by Troutt[35] explored some of these phenomena at Reynolds numbers which are ten times higher than those used in this study.

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